ANALYSES OF BARE-TETHER SYSTEMS AS A THRUSTER FOR MXER STUDIES

G. V. Khazanov, E.N. Krivorutsky, and K. Sorensen

NASA Marshall Space Flight Center, Huntsville, AL 35805(256)961-7517, (256)961-7215(fax)
George.V.Khazanov@nasa.gov

Abstract

The concept of electrodynamic tether propulsion has a number of attractive features and has been widely discussed for different applications. A number of system designs have been proposed and compared during the last ten years. In spite of this, the choice of the proper design for a specific mission is far from evident. Such characteristics of tether performance as system acceleration, efficiency, etc. should be calculated and compared based on the known capability to collect electrical current. Different designs for the Momentum-eXchange/Electrodynamic Reboost (MXER) tether concept are presented, along with the necessary electrical potential and collected current levels. Corrections to the OML (Orbital Motion Limited) current due to the tether cross-section geometry and the magnetic field, produced by the tether current, are taken into account.

1. Introduction

Electrodynamic propulsion based on the interaction of a conducting tether with the background magnetic field can be implemented across a range of system designs. Bare tethers [*Sanmartin et al.,* 1993; *Estes et al*., 2002], bare and insulated tethers with a balloon termination [*Vannaroni et al.,* 2000; *Sanmartin et al.,* 2001; *Ahedo* and *Sanmartin,* 2002], and insulated tethers with a grid-sphere termination [*Stone et al.,* 2002] have been suggested for different applications. An electrodynamic tether as a thruster is currently proposed for the Momentum-eXchange/Electrodynamic Reboost (MXER) tether facility that has the potential to provide a fully-reusable in-space propulsion infrastructure and dramatically reduce propulsion cost for many space missions [*Hoyt*, 2000; *Sorensen,* 2001].

In order for the tether system to boost multiple payloads, it must have a capability to restore its orbital energy and momentum after each payload transfer operation as rapidly as possible. The tether system that is positively-biased relative to the ambient environment require an anode contactor that will able it to collect electrons from ionospheric plasma. Because active plasma contactors require expenditure of propellant and may require significant additional mass, the bare wire tether technology is also being considered for the MXER tether [*Hoyt et al.*, 2003]. However *Stone et al.* [2002] expressed a concern about self-shielding of the bare tether at the high current levels due to the formation of a magnetic field around the wire [*Khazanov et al*., 2000, *Sanmartin* and *Estes*, 2002]. Such field may have an impact on operation of tethers that flow large currents, like current about 50 amperes, required in MXER concept.

The choice of a tether design for a specific mission is based on the analysis of tether system performance. Different parameters describing tether performance, such as system acceleration and efficiency can be calculated, if the current distribution along the tether at the satellite trajectory is known. Below we present the results of current calculation along the expected trajectory of the MXER tether for wire and tape tethers with different cross-sections, and discuss the magnetic shielding effects based on the results of *Khazanov et al.* [2001], *Sanmartin* and *Estes* [2002] papers, and the results of these calculations.

2. Tether current collection model and collected currents

The basic physics of bare tether current collection is well known**.** The detailed formulation of the problem is presented by *Sanmartin et al.* [1993], *Khazanov et al.* [2000, 2001], and *Sanmartin and Estes* [2002].

In the tether's reference frame of rest the electric field is $\vec{E} = \vec{u} \times \vec{B}_o / c$, where $\vec{u}$ is the tether orbital velocity, c is the speed of light, and B_0 is the ambient magnetic field. Below z-axe is chosen along the tether. The undisturbed potential, V_p, in plasma in the direction parallel to the wire is proportional to the projection of the electric field $\vec{E}$ on the tether direction, E_m, and $dV^p \sim E_m dz$. The potential along the wire, V^t, changes in accordance with Ohm's law. Then for the local bias we have $\Delta V = V^t - V^p$ and for the thruster mode the local bias changes as

$$\frac{d\Delta V}{dz} = \frac{I}{\sigma S} + E_m \tag{1}$$

where σ is the tether conductivity and S is the tether cross-section. The equation for the OML current I along the tether is

$$\frac{dI}{dz} = en_e \alpha(l) G \frac{p}{\pi}\sqrt{\frac{2\Delta V}{m_e}} \tag{2}$$

Here: n_e is the unperturbed electron density; G and $\alpha(l)$ are the correction multiples accounting for the deviation from the standard OML theory due to large size and cross-section geometry of the tether (G) and the self-induced magnetic field ($\alpha(l)$); p is the tether perimeter. Both coefficients, $\alpha(l)$ and G are dependent on the system parameters, but the dependence on the tether length (local bias) in G can be neglected [*Sanmartin and Estes,* 1999; *Estes and Sanmartin,* 2000]. It can be found from these papers that for a wire with the radius r_w, $r_w < \lambda_D, G = 1; r_w = 2\lambda_D, G = 0.97$. For a tape with the width w, $w \le 4\lambda_D, G = 1$; $w = 8\lambda_D, G = 0.97$; $w = 12\lambda_D, G = 0.92$; $w = 16\lambda_D, G = 0.8$. Here λ_D is the Debye length. Calculation of $\alpha(l)$ is presented by *Khazanov et al.* [2001]. In the dimensionless variables

$$l = \frac{z}{L^*};\quad L^* = \left[\frac{m_e E_m}{2e}\left(\frac{3\pi\sigma S}{4Gepn_e}\right)^2\right]^{1/3};\quad i = \frac{I}{\sigma S E_m};\quad \varphi = \frac{\Delta V}{E_m L^*} \tag{3}$$

equations (1) and (2) can be written as

$$\frac{di}{dl} = \frac{3}{4}\alpha(l)\sqrt{\varphi};\qquad \frac{d\varphi}{dl} = i + 1 \tag{4}$$

The boundary condition for the current is $i_A = 0$ at the tether end A. It is also assumed that the potential at this point, $\varphi_A = 0$, neglecting by its fluctuations in this preliminary consideration. Such approach essentially simplifies the system analyses and it is reasonable on this stage of MXER tether project. Because the solution of the equations is completely determined by the boundary conditions at point A, the results describe the fully bare tether as well as the partly insulated one.

It is expected that the MXER tether would operate in an equatorial elliptical orbit with the perigee in the altitude range of 300-500km and the apogee in the range of 5000-10000km [*Sorensen*, 2003]. The specific orbit chosen would be a function of the tip velocity of the tether, which is in turn a function of the orbital transfer desired and the limitations of tensile material strength.

Table 1

H, km	B_0, G	E_m, V/km	n_e, cm^{-3}, day	n_e, cm^{-3}, night
300	0.27	230	$1.6\cdot10^6$	$1.0\cdot10^5$
400	0.26	220	$1.5\cdot10^6$	$3.0\cdot10^5$
500	0.25	210	$9.0\cdot10^5$	$2.0\cdot10^5$
700	0.23	190	$2.0\cdot10^5$	$8.0\cdot10^4$
900	0.21	170	$7.0\cdot10^4$	$3.0\cdot10^4$

The induced electric field for such a trajectory in the range of altitudes 300-900km is presented in Table 1. The angle between the satellite velocity and the Earth's magnetic field has been taken 90º in these calculations. Table 1 also contains the typical plasma densities for day and night. The temperature is taken to be 1900º K for all calculations presented below. The range of the cross-sections and the tethers length have been chosen taking into account the currents required for the MXER tether facility.

The current is calculated for the wires with the radiuses, r_w, equal 2.0mm, 2.5mm, and 3.0mm. For the same plasma parameters the current and local bias are also calculated for three tapes with the cross-sections presented in the Table 2, neglecting the self-induced magnetic field. The self-induced magnetic field effect for the tape should be essentially smaller than for the wire because of the topology of the magnetic field around a tape. The same conclusion follows from the analyses of *Sanmartin,and Estes* [2002].

Table 2

Cross-section area, mm^2	Width, mm	Thickness, mm
4.0	20.0	0.2
4.0	40.0	0.1
6.0	60.0	0.1

Examples of the current and local bias distributions along the tether are presented in Fig.1. The length at these plots is the length of the bare part of the tether, counted from the end of the tether.

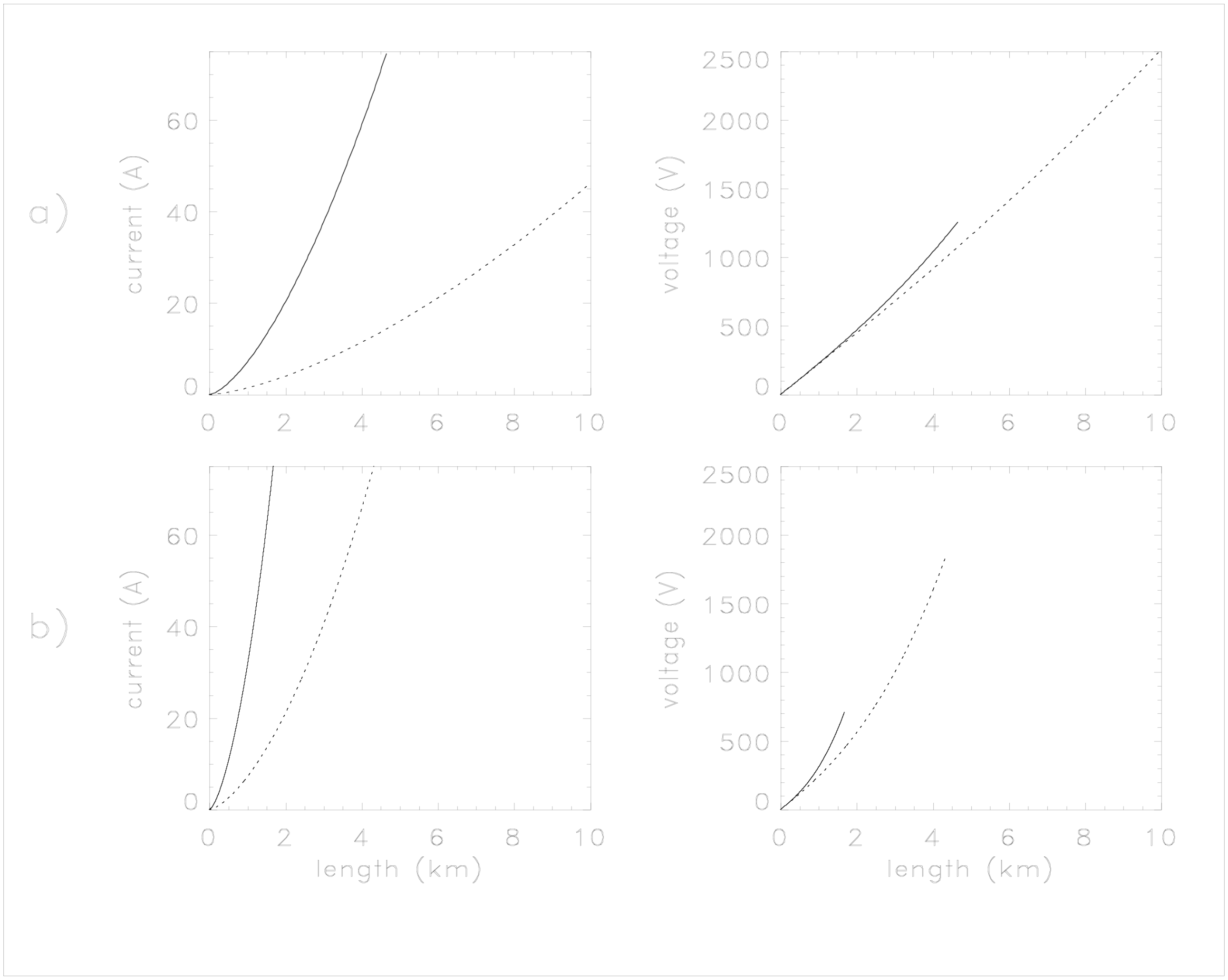

Figure 1. Current and local bias distributions along the tether for the altitude 400km. Panel a) is for the wire with radius 2.5mm; panel b) is for the tape with the cross-section $4mm^2$ and the tape width 40mm. Solid and dotted lines are for day and night plasma densities respectively.

Currently investigated designs for the MXER tether facility requires current levels of about 50A. Table 3 contains the tether length (L) and the voltage (V) needed for collection of such current for tethers with different geometry (wire radius, r_w, and tape width, w) at different altitudes (H). Subscripts 'd' and 'n' corresponds to day and night. The tether length is restricted by 20km for the wire and by 10km for the tape.

Table 3

H, km	300			400			500			700			900	
r_w,mm	**2.0**	**2.5**	**3.0**	**2.0**	**2.5**	**3.0**	**2.0**	**2.5**	**3.0**	**2.0**	**2.5**	**3.0**	**2.5**	**3.0**
L_d,km	4.0	3.3	3.0	4.0	3.6	3.0	6.0	5.0	4.5	16.5	15.0	12.7	>20	>20
V_d,kV	1.1	0.9	0.8	1.1	0.9	0.75	1.6	1.2	1.05	4.0	3.3	2.6		
L_n,km	>20	>20	19	12.0	10.5	9.5	16.0	14.0	12.5	>20	>20	>20	>20	>20
V_n,kV			5.0	3.3	2.8	2.3	4.2	3.4	2.9					
w,cm	**2.0**	**4.0**	**6.0**	**2.0**	**4.0**	**6.0**	**2.0**	**4.0**	**6.0**	**2.0**	**4.0**	**6.0**	**4.0**	**6.0**
L_d,km	1.6	1.2	1.3	1.8	1.2	1.3	2.5	1.6	1.5	7.1	4.5	3.6	9.5	7.5
V_d,kV	0.6	0.45	0.4	0.65	0.45	0.4	0.8	0.6	0.45	2.2	1.4	1.0	2.9	2.0
L_n,km	>10	6.8	5.5	5.3	3.5	2.8	7.0	4.5	3.5	>10	8.4	6.5	>10	>10
V_n,kV		2.6	1.8	1.9	1.3	0.9	2.4	1.6	1.05		2.7	1.8		

Calculated results are covering a wide range of plasma and tether parameters along the planned MXER tether trajectory and can be used for the preliminary analyses of the tether performance and choice of the preferable technology for system restoration.

3. Discussion

Because the currents required for MXER tether operation are large and their magnetic fields are large compared to the ambient magnetic field in the vicinity of the tether we will discuss here the problem of magnetic shielding in some details. As can be seen from equation (4) setting the coefficient □(l) equal to one eliminates the current's magnetic field from the calculations. Current found in this approximation is practically the same as presented above and the magnetic shielding is negligible for the equatorial elliptical orbit. It should be stressed that it is not always the case and essentially depends on the specific of the mission. If an inclined orbit for the MXER tether were to be used then the role of magnetic shielding can be not negligible. Equatorial orbit is strongly favored for the tether since it allows repeated rendezvous opportunities between the tether and payload in the event of a missed "catch". Figure 2 presents the current reduction for the thruster mode for a tether on a non-equatorial orbit.

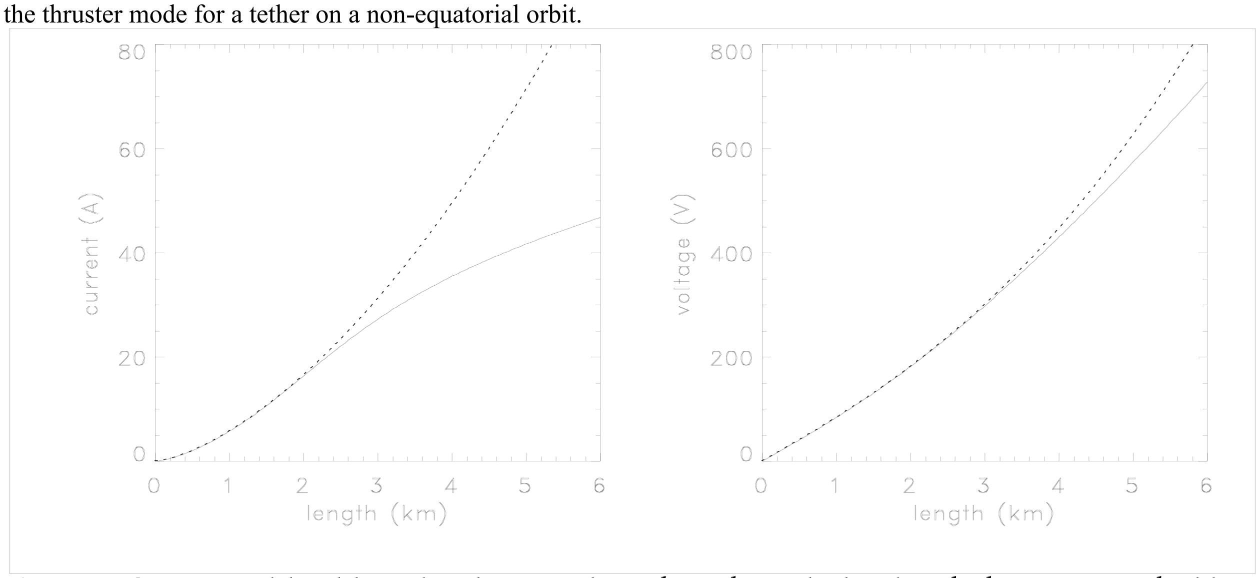

Figure 2. Current and local bias distributions along the tether calculated with the magnetic shielding (solid lines), and neglecting it, for non-equatorial trajectory; r_w=2.5mm, n_e=$2\cdot10^6$cm^{-3}, E_m=80V/km, B_0=0.25G

The angle between the tether and the ambient magnetic field in this case is taken not 90º, as in the calculations above, but 30º. As can be seen from the Figure 2 for such trajectory the effect of shielding is essential. The current is reduced by a factor of two for the tether with a bare length of 5km. The reason for such different role of magnetic shielding for these two orbits is discussed below.

Results of this study are based on the model of magnetic shielding of *Khazanov et al*. [2000, 2001]. Below we present the comparison of this model with the model of magnetic shielding developed by *Sanmartin and Estes* [2002].

The electrodynamics of a long conducting bare or partly insulated tether is based on the OML theory of a cylindrical probe [*Laframboise,* 1966]. If tether current is large, the induced magnetic field will affect the current collection and change the OML relation between the current and the local bias $dI(z)/dz \sim \sqrt{\Delta V(z)}$ along the tether. The tether current produces a closed, azimuthal magnetic field around the tether. As a result, the region immediately surrounding the tether is disconnected from the open magnetic field region farther out (a magnetic separatrix exists). Therefore in order to be collected, charged particles must intersect the boundary surface (separatrix) between the regions of closed and open magnetic fields configurations. If the plasma sheath is inside the region of closed magnetic surfaces, the particle can be collected only due to the thermal motion, i.e. finite Larmour radius. To the extend that charged particles are unable to move across these surfaces, collected current will be reduced. This magnetic insulation breaks down if the boundary surface is inside the region of strong electric field, i.e. inside the plasma sheath. In this case the collected current is just the OML current. The relationship of these two surfaces to each other changes along the tether and depends on the system parameters. It can be assumed that the plasma sheath and the boundary between the closed and open regions of magnetic field coincide, if their lengths are equal. Therefore, for that part of the tether where the length of the plasma sheath is larger than the length of the separatrix, the collected current can be calculated as OML current [*Khazanov et al.,* 2000, 2001]. In the opposite case the current should be calculated from the thermal flux.

The reason for so different role of magnetic shielding for the equatorial orbit and the non-equatorial orbit described above is because of the strong dependence of the separatrix length on the angle between the current and the ambient magnetic field. If the ambient magnetic field is parallel to the wire, the total magnetic field is a set of cylinders parallel to the wire, and with the neglected end-flux the electron transport to the wire is across the magnetic field. When there is some angle between the current and the ambient magnetic field the region of closed cylinders around the wire is restricted by some radius, and till this radius the electron transport is along the magnetic field lines and only for smaller radius the transport is across the magnetic field. The region of closed magnetic surfaces shrinks as the angle between the current and the ambient magnetic field increases reaching the minimum for equatorial orbit. Obviously it is true for a fixed current magnitude while this magnitude is also angle dependent. But the tendency still prevails and the magnetic shielding affect is smallest for the 90º angle between the current and ambient magnetic field. In more details this is discussed by *Khazanov et al*. [2000, 2001].

Sanmartin and *Estes* [2002] proposed a similar model of magnetic shielding as *Khazanov et al*. [2000, 2001] with two changes. They neglect the current due to thermal motion of particles across the separatrix by assuming that the collected current is zero, if the separatrix is inside the plasma sheath. This leads to overestimation of the effect. They also use a different model of the plasma sheath, with the radius approximately 1.34 times larger compare to that used by *Khazanov et al.* [2000, 2001]. This reduces the influence of the induced magnetic field on current collection.

Sanmartin and *Estes* [2002] also introduced restrictions on the system parameters based on the prescribed efficiency, tether length, and orbital inclination. These restrictions however should be instead mission specific, rather than generally imposed. The significance of this is illustrated below, based on the example from *Sanmartin* and *Estes* [2002] for a tether operating in the generator mode. In section 4.1 of this paper they calculate the current reduction due to the self-induced magnetic field for a 20km long tether. The tether radius is taken to be 1.81mm, plasma density is $2{\cdot}10^6$cm^{-3}, and the electric field is 100V/km. The electrical load is 5 times the tether resistance. The efficiency for such load is ~0.8 and the current reduction is 0.14%. *Khazanov et al.* [2001] also found that for such parameters the current reduction is negligible. It was found that even for a load two times larger than the tether resistance the effect is small (see *Khazanov et al.* [2001], Figure 7). As a consequence *Khazanov et al.* [2001] restricted their analyses to the case where the maximum power is sought, i.e. the load and tether resistances are equal. For such a load the current reduction in the example from *Sanmartin* and *Estes* [2002] will be ~10%. It should be noted that for this case *Khazanov et al.* [2001] derived a smaller current reduction.

We also compared the current reduction from these two models for the drag mode for a tether 20km long with the radius 1.8mm. For plasma density $2{\cdot}10^6$cm^{-3}, and electric field 100V/km, *Sanmartin* and *Estes* [2002] in Section 4.2 found that the current reduction is 17%; for the same tether, the plasma density $1.7{\cdot}10^6$cm^{-3}, and the electric field 130V/km, the reduction is 6% in the Table 3 from *Khazanov et al.* [2001].

The results from these two papers can not be directly compared for the thruster mode because of the difference in the boundary conditions, but the dependence of current reduction from different system parameters is similar.

As mentioned above, *Sanmartin* and *Estes* [2002] and *Khazanov et al.* [2001] adopted different models for the plasma sheath size. The effect of this difference is illustrated in Figure 3 for the system parameters same as in the example presented by *Sanmartin* and *Estes* [2002] in Section 5 of their paper for the thruster mode. We inserted in our model the plasma sheath radius 1.34 times larger, according to the plasma sheath size in Section 2, *Sanmartin* and *Estes* [2002]. As can be seen from Figure 3 the self-induced magnetic field manifests starting with a current of 40A, and a tether bare length of 2.6km for the case with larger plasma sheath radius as compared to 30A and 2.0km in the case of smaller plasma sheath.

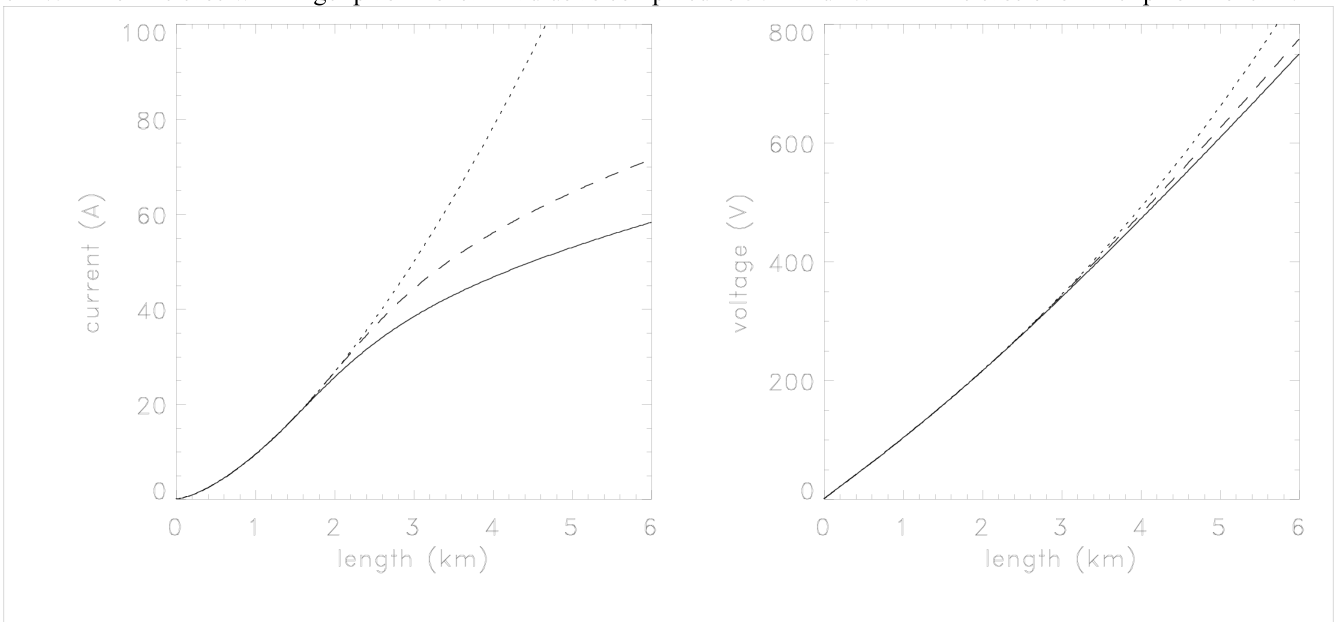

Figure 3. Current and local bias distributions along the tether calculated with the magnetic shielding (solid lines for plasma sheath model from *Khazanov et al.* [2001], dashed lines for plasma sheath model from *Sanmartin* and *Estes* [2002]), and neglecting it. System parameters are: r_w=3.66mm, $n_e=2\cdot10^6 cm^{-3}$, E_m=100V/km, angle between the current and magnetic field 35°.

It should be stressed that the paper of *Khazanov et al.* [2001] only outlines the boundaries of the plasma-tether system parameters where the effect of self-induced magnetic field can be expected, while *Sanmartin* and *Estes* [2002] studied the effect under restrictions on the system parameters, based on the prescribed efficiency, tether length, and trajectory inclination. When the system parameters in both models are the same the numerical results of both models are close. (In *Khazanov et al.* [2001], in their example in the abstract, are two misprints: the density should be $6.68\cdot10^6 cm^{-3}$ and the angle between the Earth's magnetic field and the horizontal plane should be 70^0. Correct data are presented at their Fig. 13.) Evidently some difference is introduced by the difference in the plasma sheath size, and the way in which the shielding is inserted in the models, as has been discussed above. Some difference results also from the analytical approximations used by *Sanmartin* and *Estes* [2002]. Both models gave similar dependence of current reduction on the system parameters, such as the plasma density, tether length, etc., and tether operation modes.

4. Conclusion

We calculated the current that can be collected by wire and tape tethers (bare or partly insulated) along an equatorial elliptical orbit with the perigee in the altitude range of 300-500km and the apogee between 5000-10000km. Calculations are performed for the day and night conditions. Such orbital conditions are expected for the MXER tether operational concept. The current is calculated taking into account the corrections to the Orbit Limited Model that result from the size and the form of tether cross-section and magnetic shielding. It has been found that for the planned equatorial orbit the effect of magnetic shielding is negligible, while it can be essential for orbits with significant inclination. We also compared the known models describing the magnetic shielding. It has been found that both models gave close results for the similar system parameters. Both models gave also similar dependence of current reduction on the system parameters, such as the plasma density, tether length, etc., and tether operation modes.

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